Proving a Quadrilateral is a Parallelogram

Method 1: Show that the diagonals bisect each other by showing the midpoints of the diagonals are the same

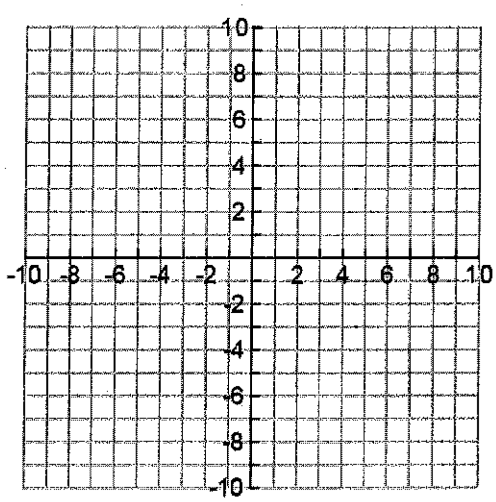
Method 2: Show both pairs of opposite sides are parallel by showing they have equal slopes.

Method 3: Show both pairs of opposite sides are equal by using distance.

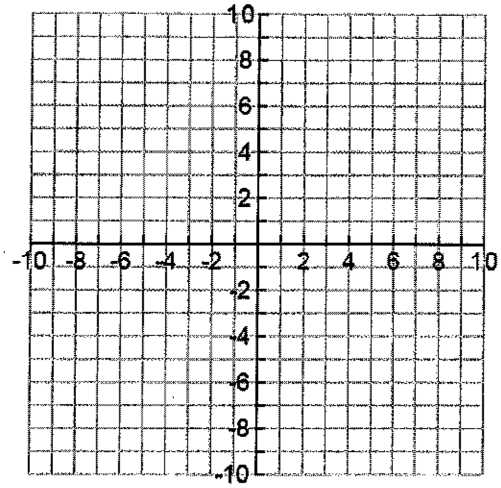
Method 4: Show one pair of sides is both parallel and equal.

Examples

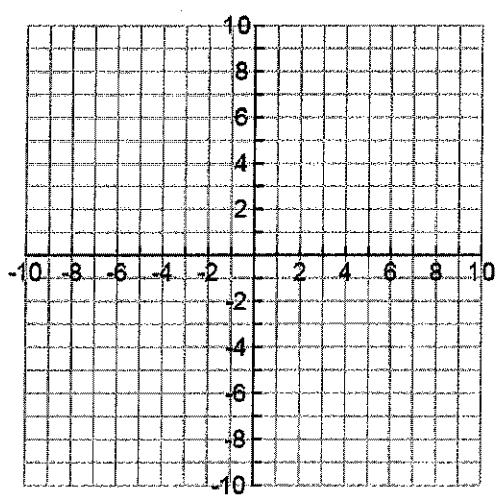
1. Prove that the quadrilateral with the coordinates L(-2,3), M(4,3), N(2,-2) and O(-4,-2) is a parallelogram.



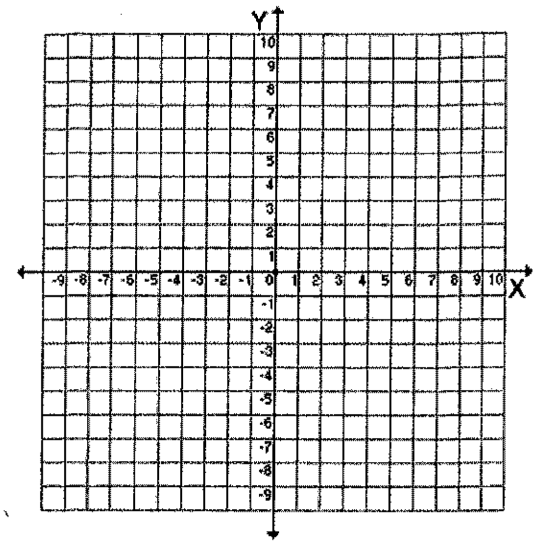
2. Prove that the quadrilateral with the coordinates P(1,1), Q(2,4), R(5,6) and S(4,3) is a parallelogram.



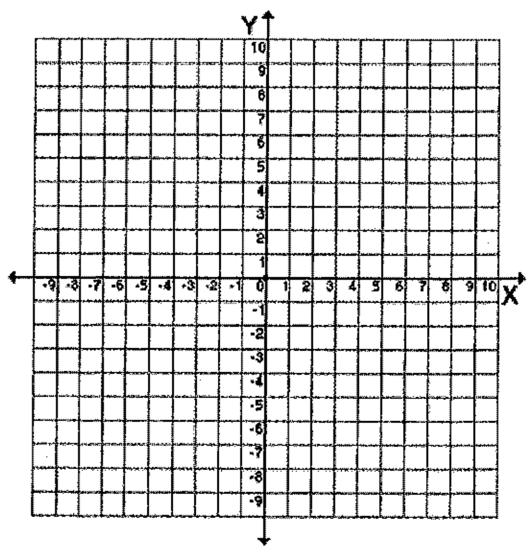
3. Prove that the quadrilateral with the coordinates R(3,2), S(6,2), T(0,-2) and U(-3,-2) is a parallelogram.



5. The vertices of quadrilateral *JOHN* are $J(-3, 1)$, $O(3, 3)$, $H(5, 7)$, and $N(-1, 5)$. Use coordinate geometry to prove that quadrilateral *JOHN* is a parallelogram.



6. Prove that quadrilateral LEAP with the vertices L(-3,1), E(2,6), A(9,5) and P(4,0) is a parallelogram.



– Using Coordinate Geometry to Prove Rectangles, Rhombi, and Squares

Proving a Quadrilateral is a Rectangle

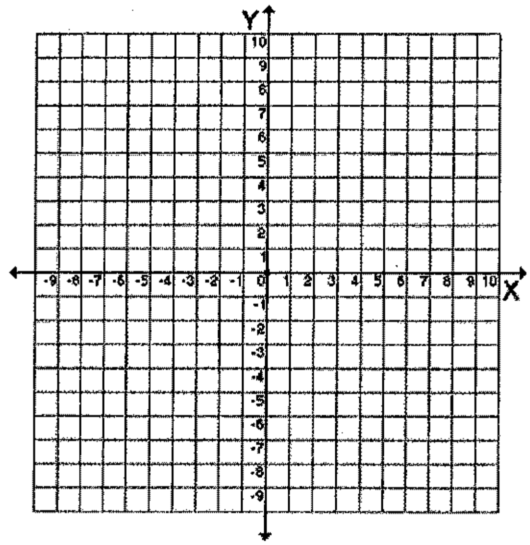
Prove that it is a parallelogram first, then:

Method 1: Show that the diagonals are congruent.

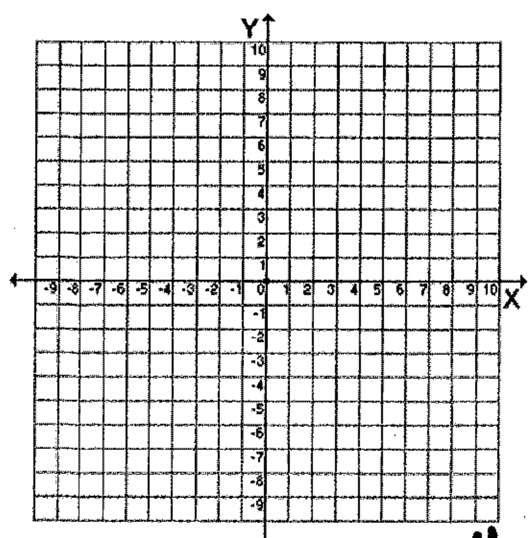
Method 2: Show that it has a right angle by using slope.

Examples:

1. Prove a quadrilateral with vertices G(1,1), H(5,3), I(4,5) and J(0,3) is a rectangle.



2. The vertices of quadrilateral COAT are C(0,0), O(5,0), A(5,2) and T(0,2). Prove that COAT is a rectangle.



Proving a Quadrilateral is a Rhombus

Prove that it is a parallelogram first, then:

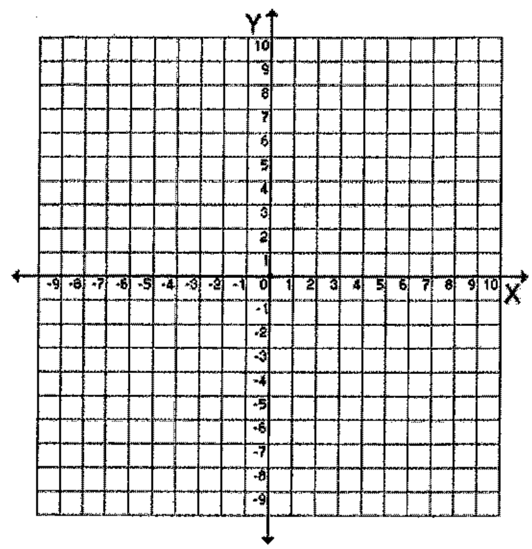
Method 1: Prove that the diagonals are perpendicular.

Method 2: Prove that a pair of adjacent sides are equal.

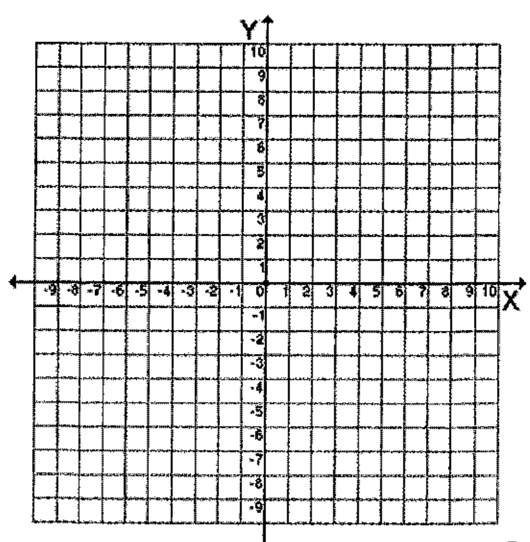
Method 3: Prove that all four sides are equal.

Examples:

1. Prove that a quadrilateral with the vertices A(-2,3), B(2,6), C(7,6) and D(3,3) is a rhombus.



2. Prove that the quadrilateral with the vertices A(-1,4), B(2,6), C(5,4) and D(2,2) is a rhombus.

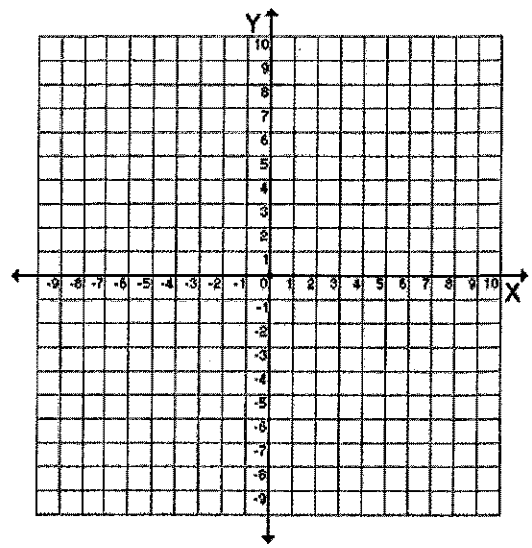


Proving that a Quadrilateral is a Square

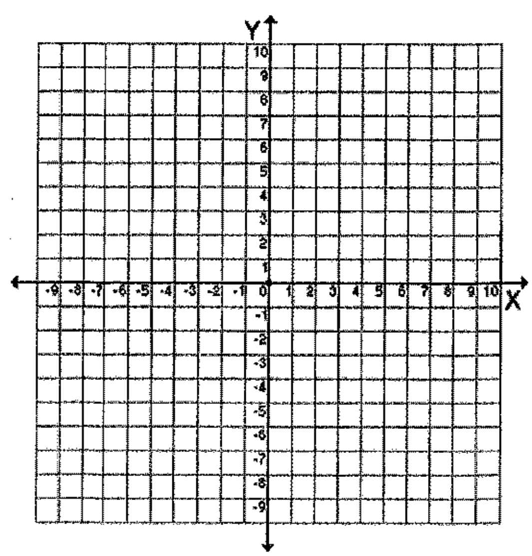
There are many ways to do this. I recommend proving the diagonals bisect each other (parallelogram), are equal (rectangle) and perpendicular (rhombus).

Examples:

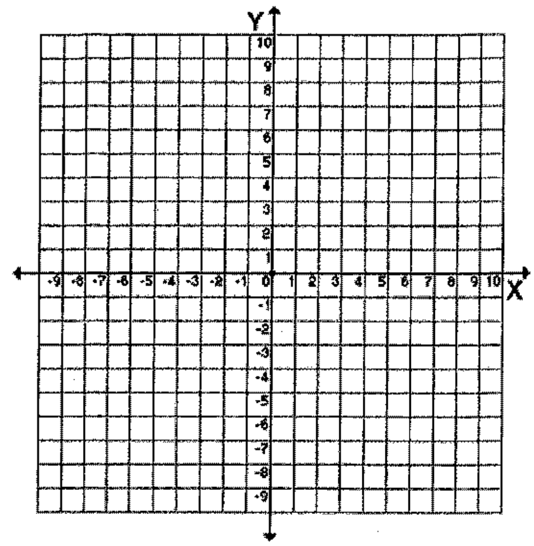
1. Prove that the quadrilateral with vertices A(0,0), B(4,3), C(7,-1) and D(3,-4) is a square.



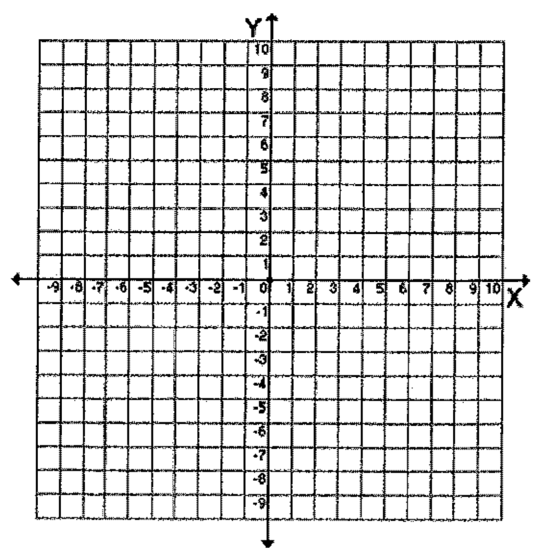
2. Prove that the quadrilateral with vertices A(2,2), B(5,-2), C(9,1) and D(6,5) is a square.



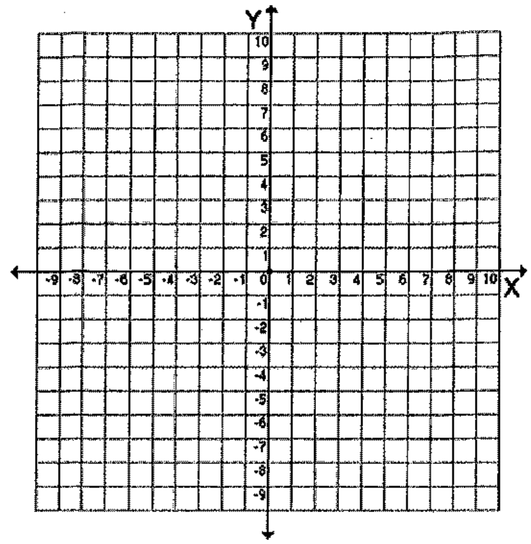
1. Prove that quadrilateral ABCD with the vertices A(2,1), B(1,3), C(-5,0), and D(-4,-2) is a rectangle.



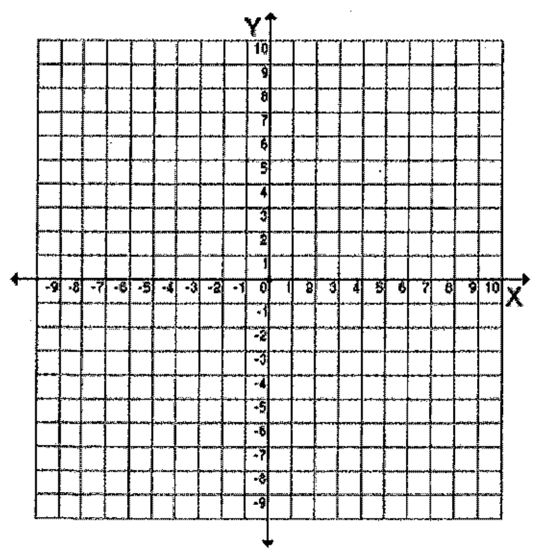
2. Prove that quadrilateral PLUS with the vertices P(2,1), L(6,3), U(5,5), and S(1,3) is a rectangle.



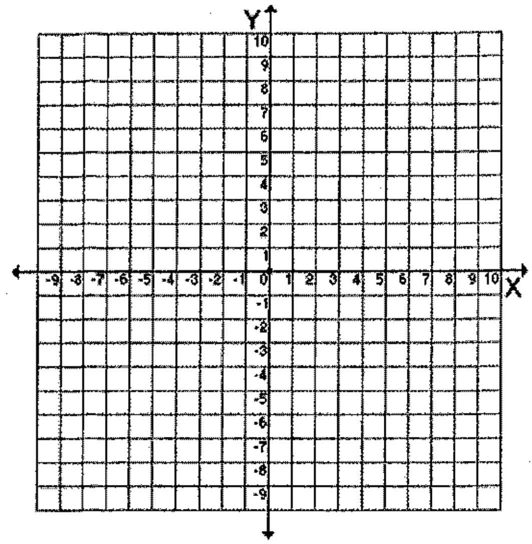
3. Prove that quadrilateral DAVE with the vertices D(2,1), A(6,-2), V(10,1), and E(6,4) is a rhombus.



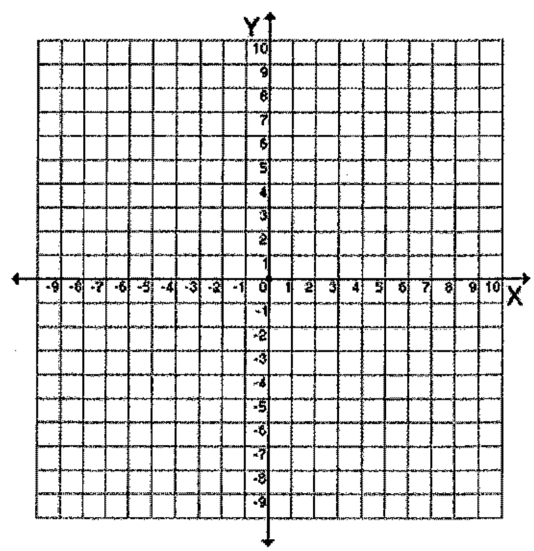
4. Prove that quadrilateral GHIJ with the vertices G(-2,2), H(3,4), I(8,2), and J(3,0) is a rhombus.



5. Prove that a quadrilateral with vertices J(2,-1), K(-1,-4), L(-4,-1) and M(-1, 2) is a square.



6. Prove that ABCD is a square if A(1,3), B(2,0), C(5,1) and D(4,4).



– Using Coordinate Geometry to Prove Trapezoids

Proving a Quadrilateral is a Trapezoid

Show one pair of sides are parallel (same slope) and one pair of sides are not parallel (different slopes).

Proving a Quadrilateral is an Isosceles Trapezoid

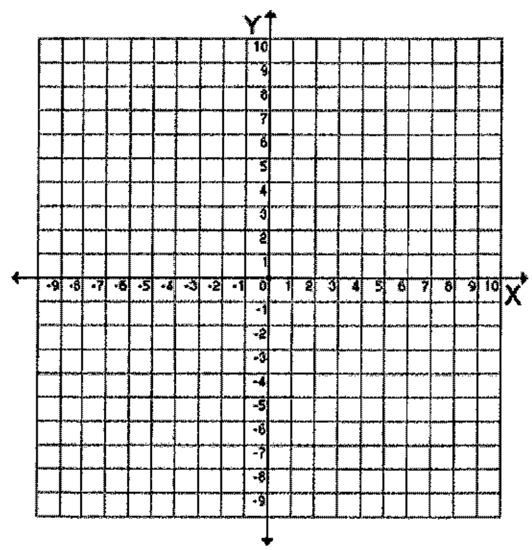
Prove that it is a trapezoid first, then:

Method 1: Prove the diagonals are congruent using distance.

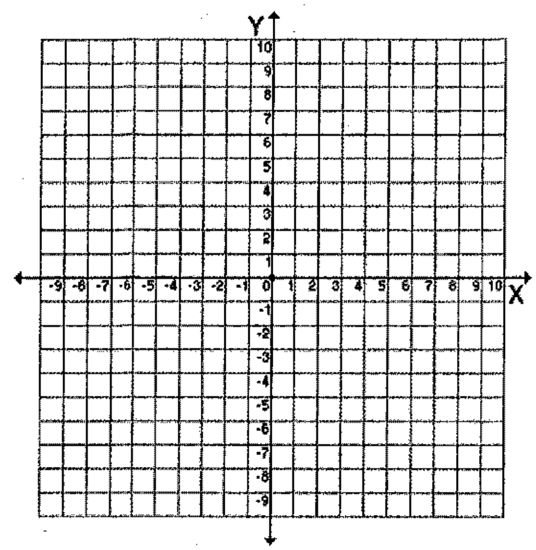
Method 2: Prove that the pair of non parallel sides are equal.

Examples:

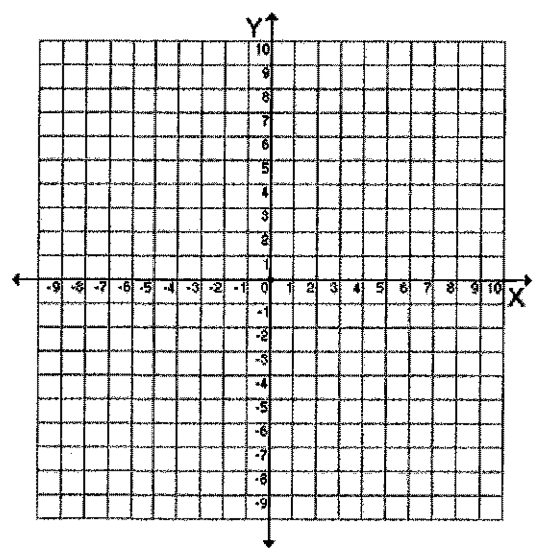
1. Prove that KATE a trapezoid with coordinates K(1,5), A(4,7), T(7,3) and E(1,-1).



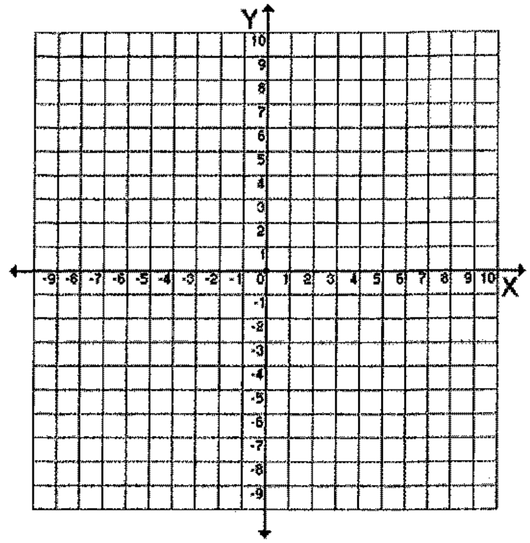
2. Prove that quadrilateral MILK with the vertices M(1,3), I(-1,1), L(-1, -2), and K(4,3) is an isosceles trapezoid.



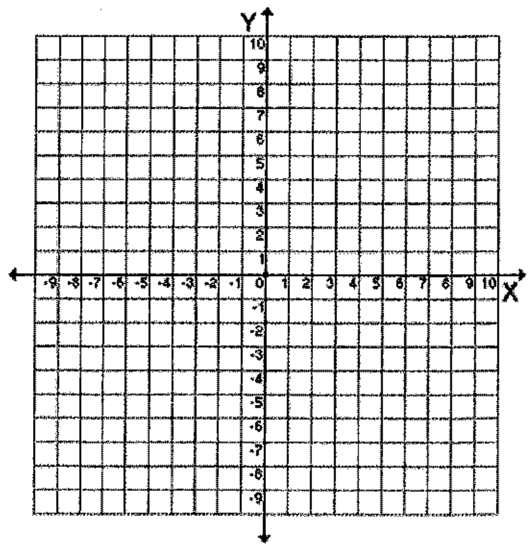
3. Prove that the quadrilateral with the vertices C(-3,-5), R(5,1), U(2,3) and D(-2,0) is a trapezoid but not an isosceles trapezoid.



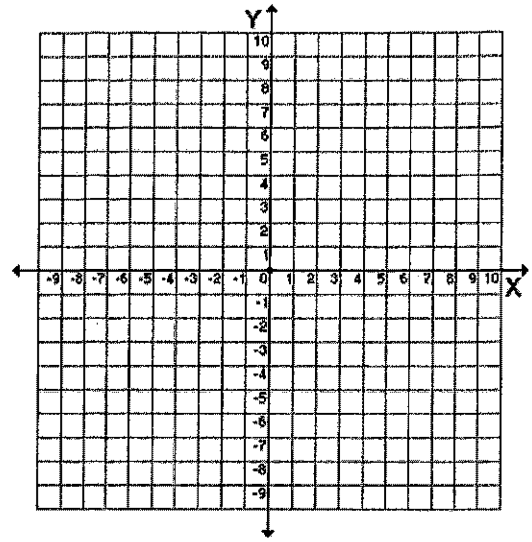
1. The vertices of quadrilateral *MARY* are *M*(−3, 3), *A*(7, 3), *R*(3, 6), and *Y*(1, 6). Use coordinate geometry to prove that quadrilateral *MARY* is an isosceles trapezoid.



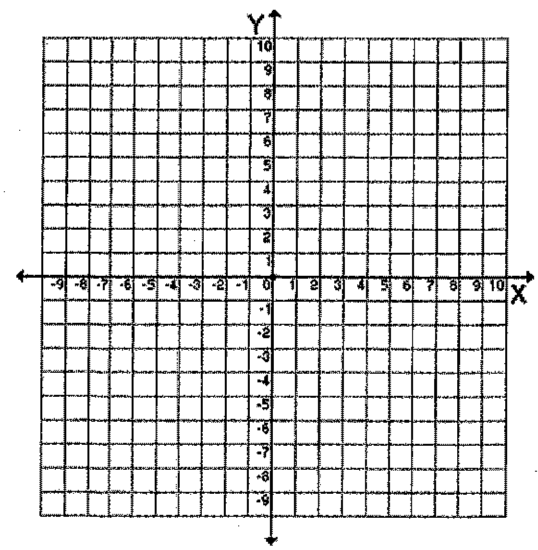
2. Quadrilateral *JACK* has vertices *J*(1, −4), *A*(10, 2), *C*(8, 5), and *K*(2, 1). Use coordinate geometry to prove that
   a. quadrilateral *JACK* is a trapezoid.
   b. quadrilateral *JACK* is *not* isosceles.



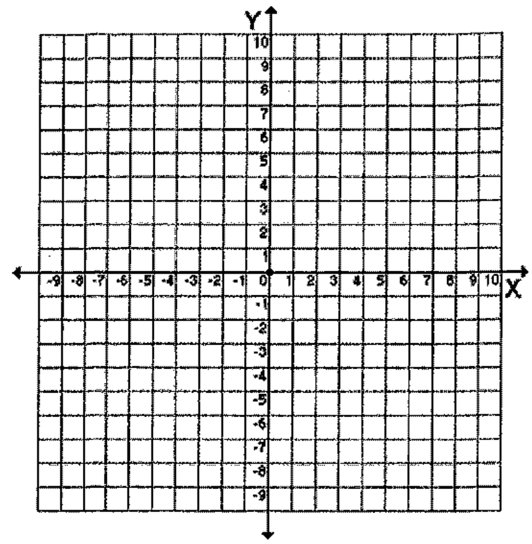
3. Quadrilateral ABCD has vertices A(-6,3), B(-3,6), C(9,6) and D(-5,-8). Prove that quadrilateral ABCD is:
   a. a trapezoid
   b. not an isosceles trapezoid



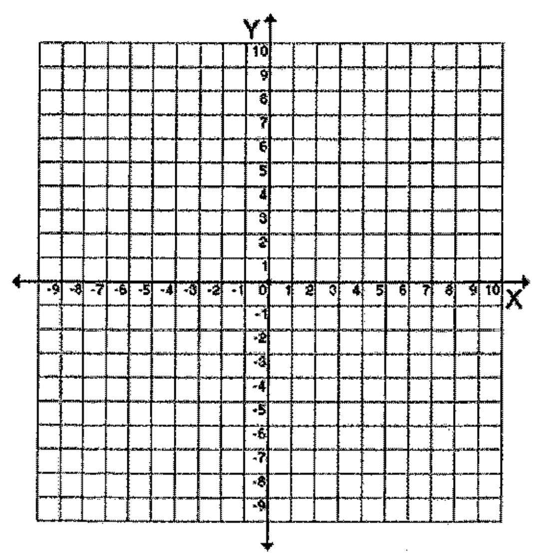
4. The vertices of quadrilateral ABCD are A(-3,-1), B(6,2), C(5,5) and D(-4,2). Prove that quadrilateral ABCD is a rectangle.



5. The vertices of quadrilateral ABCD are A(-3,1), B(1,4), C(4,0) and D(0,-3). Prove that quadrilateral ABCD is a square.



6. Quadrilateral METS has vertices M(-5, -2), E(-5,3), T(4,6) and S(7,2). Prove by coordinate geometry that quadrilateral METS is an isosceles trapezoid.



SUMMARY

If you are asked to prove:	Suggestions on how to do this:
Two lines parallel	*Use* the slope formula twice. (Find the slopes of the two lines.) *Determine* that the slopes are equal, therefore the lines are parallel.
Two lines perpendicular	*Use* the slope formula twice. (Find the slopes of the two lines.) *Determine* that the slopes are negative reciprocals of each other, therefore the lines are perpendicular.
A triangle is a right triangle	*Use* the slope formula twice. (Find the slopes of the legs.) *Determine* that since the slopes are negative reciprocals of each other, the lines are perpendicular, forming a right angle. This makes the triangle a right triangle. OR *Use* the distance formula three times. (Find the lengths of the three sides.) *Determine* that the square of the length of the hypotenuse (the longest side) is equal to the sum of the squares of the lengths of the two adjacent legs, that is, use the Pythagorean theorem: ($c^2 = a^2 + b^2$).
A triangle is isosceles	*Use* the distance formula twice. (Find the lengths of two congruent sides.) *Determine* that since the lengths of two sides are equal, the triangle is isosceles.
A triangle is an isosceles right triangle	*Use* the distance formula three times. (Find right triangle the lengths of the three sides.) *Determine* that since the lengths of two sides are equal and that the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two adjacent legs ($c^2 = a^2 + b^2$), the triangle is an isosceles right triangle. OR *Use* the slope formula twice and the distance formula twice. (Find the slopes and the lengths of the two legs.) First, prove the triangle is a right triangle (see above), and then use the distance formula to find the lengths of the two legs of the triangle. Since the lengths of two sides are equal, the triangle is isosceles. Thus, the triangle is an isosceles right triangle.
A quadrilateral is a parallelogram	*Use* the slope formula four times. (Find the slopes of the four sides.) *Determine* that since the slopes of both pairs of opposite sides are equal, which makes both pairs of opposite sides parallel, the quadrilateral is a parallelogram. OR *Use* the distance formula four times. (Find the lengths of the four sides.) *Determine* that since the lengths of both pairs of opposite sides are equal, the quadrilateral is a parallelogram. OR *Use* the slope formula twice and the distance formula twice (on the same pair of opposite sides).

If you are asked to prove:	Suggestions on how to do this:
A quadrilateral is a parallelogram	*Determine* that since the slopes of one pair of opposite sides are equal, the sides are parallel. Also determine that the same pair of opposite sides have equal lengths. Since one pair of opposite sides is both parallel and equal, the quadrilateral is a parallelogram. OR *Use* the midpoint formula twice. (Find the midpoints of the diagonals.) *Determine* that the midpoints of the diagonals are the same. Thus, the diagonals bisect each other and the quadrilateral is a parallelogram.
A quadrilateral is a rectangle	*Use* the slope formula four times. (Find the slopes of the four sides.) *Determine* that since the slopes of each pair of adjacent sides are negative reciprocals, the lines are perpendicular, which form right angles. A quadrilateral with four right angles is a rectangle. OR First prove that the quadrilateral is a parallelogram. Then use the distance formula twice. (Find the lengths of the diagonals.) *Determine* that the parallelogram has congruent diagonals, which makes it a rectangle. OR First prove that the quadrilateral is a parallelogram. Then use the slope formula twice. (Find the slopes of two adjacent sides.) *Determine* that the slopes of the sides are negative reciprocals, of each other, which show that they are perpendicular. Thus, the parallelogram has one right angle, which makes it a rectangle.
A quadrilateral is a rhombus	*Use* the distance formula four times. (Find the lengths of the four sides.) *Determine* that the quadrilateral is equilateral, which makes it a rhombus. OR First prove the quadrilateral is a parallelogram and then *Use* the slope formula twice. (Find the slopes of the diagonals.) *Determine* that the quadrilateral is a parallelogram whose diagonals are perpendicular, which makes it a rhombus.
A quadrilateral is a square	*Use* the distance formula six times. (Find the lengths of the four sides and the diagonals.) *Determine* that the quadrilateral is a rhombus (since it is equilateral) with congruent diagonals, making it a square. OR First prove the quadrilateral is a rectangle and then *Use* the distance formula twice. (Find the lengths of two adjacent sides.) *Determine* that the quadrilateral is a rectangle with one pair of congruent adjacent sides, which makes it a square. OR First prove the quadrilateral is a rectangle and then *Use* the slope formula twice. (Find the slopes of the diagonals.) *Determine* that the quadrilateral is a rectangle whose diagonals are perpendicular, which makes it a square.
A quadrilateral is a trapezoid	*Use* the slope formula four times. (Find the slopes of the four sides.) *Determine* that one pair of opposite sides is parallel (the slopes are the same) and one pair of opposite sides is not parallel (the slopes are not the same).
A quadrilateral is an isosceles trapezoid	First prove the quadrilateral is a trapezoid and then *Use* the distance formula twice. (Find the lengths of the nonparallel sides.) *Determine* that the quadrilateral is a trapezoid whose nonparallel sides are equal, which makes it an isosceles trapezoid.