Name: ______________________________ Date: _______________

IRRATIONAL NUMBERS
COMMON CORE ALGEBRA I

The set of real numbers is made up of two distinctly different numbers. Those that are **rational** and those that are **irrational**. Their technical definitions are given below.

RATIONAL AND IRRATIONAL NUMBERS

1. A **rational rumber** is any number that can be written as the **ratio** of **two integers**. Such numbers include $\frac{3}{4}$, $\frac{-7}{3}$, and $\frac{5}{1}$. These numbers have **terminating** or **repeating decimals**.
2. An **irrational number** is any number that is **not rational**. So, ones that cannot be written as the ratio of two integers. These numbers have **nonterminating and nonrepeating decimal representations.**

***Exercise* #1:** Let's consider a number that is rational and one that is irrational (**not rational**). Consider the rational number $\frac{2}{3}$ and the irrational number $\sqrt{\frac{1}{2}}$. Both of these numbers are less than 1.

(a) Draw a pictorial representation of $\frac{2}{3}$ of the rectangle shown below.

(b) Using your calculator, give the decimal representation of the number $\frac{2}{3}$. Notice that it has a repeating decimal pattern.

(c) Write out all of the decimal places that your calculator gives you for $\sqrt{\frac{1}{2}}$. Notice that it does not have a repeating decimal pattern.

(d) Why could you not draw a pictorial representation of $\sqrt{\frac{1}{2}}$ that way you do for $\frac{2}{3}$?

Irrational numbers are necessary for a variety of reasons, but they are somewhat of a mystery. In essence they are a number that can never be found by **subdividing** an **integer quantity** into a **whole number** of **parts** and then taking an **integer number** of those parts. There are many, many types of irrational numbers, but **square roots of non-perfect squares** are **always irrational**. The proof of this is beyond the scope of this course.

***Exercise* #2:** Write out every decimal your calculator gives you for these **irrational numbers** and notice that they never repeat.

(a) $\sqrt{2}=$

(b) $\sqrt{10}=$

(c) $\sqrt{23}=$

Rational and irrational numbers often mix, as when we simplify the square root of a non-perfect square.

***Exercise* #3:** Consider the **irrational** number $\sqrt{28}$.

(a) Without using your calculator, between what two consecutive integers will this number lie? Why?

(b) Using your calculator, write out all decimals for $\sqrt{28}$.

(c) Write $\sqrt{28}$ in simplest radical form.

(d) Write out the decimal representation for your answer from (c). Notice it is the same as (b).

O.k. So, it appears that a **non-zero rational number times an irrational number** (see letter (c) above) results in an **irrational number** (see letter (d) above). We should also investigate what happens when we add rational numbers to irrational numbers (and subtract them).

***Exercise* #4:** For each of the following addition or subtraction problems, a rational number has been added to an irrational number. Write out the decimal representation that your calculator gives you and classify the result as rational (if it has a repeating decimal) or irrational (if it doesn't).

(a) $\frac{1}{2}+\sqrt{2}$

(b) $\frac{4}{3}+\sqrt{10}$

(c) $7-\sqrt{8}$

***Exercise* #5:** Fill in the following statement about the sum or rational and irrational numbers.

When a **rational number** is added to an **irrational number** the result is always ______________________.

***Exercise* #6:** Which of the following is an irrational number? If necessary, play around with your calculator to see if the decimal representation does not repeat. **Don't be fooled by the square roots**.

(1) $\sqrt{25}$

(3) $\frac{7}{2}$

(2) $4-\sqrt{9}$

(4) $3+\sqrt{6}$

Name: ____________________________________ Date: __________________

IRRATIONAL NUMBERS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. For each of the following rational numbers, use your calculator to write out either the terminating decimal or the repeating decimal patterns.

(a) $\frac{3}{4}$ (b) $\frac{4}{9}$ (c) $\frac{5}{8}$ (d) $\frac{5}{6}$

(e) $\sqrt{\frac{25}{4}}$ (f) $\sqrt{\frac{1}{100}}$ (g) $\sqrt{\frac{4}{9}}$ (h) $\sqrt{\frac{2}{32}}$

2. One of the most famous **irrational numbers** is the number pi, π, which is essential in calculating the circumference and area of a circle.

(a) Use your calculator to write out all of the decimals your calculator gives you for π. Notice no repeating pattern.

(b) Historically the rational number $\frac{22}{7}$ has been used to **approximate** the value of π. Use your calculator to write out all of the decimals for this rational number and compare it to (a).

3. For each of the following irrational numbers, do two things: (1) write the square root in simplest radical form and then (2) use your calculator to write out the decimal representation.

(a) $\sqrt{32}$ (b) $\sqrt{98}$ (c) $\sqrt{75}$

(d) $\sqrt{500}$ (e) $\sqrt{80}$ (f) $\sqrt{117}$

REASONING

Types of numbers mix and match in various ways. The last exercise shows us a trend that we explored during the lesson.

4. Fill in the statement below based on the last exercise with one of the words below the blank.

   The product of a (non-zero) rational number and an irrational number results in a(n) ____________ number.
   rational   irrational

Now we will explore other patterns in the following exercises.

5. Let's explore the **product** of **two irrational numbers** to see if it is **always irrational, sometimes irrational, sometimes rational,** or **always rational.** Find each product below using your calculator (be careful as you put it in) and write out all decimals. Then, classify as either rational or irrational.

   (a) $\sqrt{5}\cdot\sqrt{3}=$ ______________________ Rational or irrational?

   (b) $\sqrt{8}\cdot\sqrt{18}=$ ______________________ Rational or irrational?

   (c) $\sqrt{7}\cdot\sqrt{11}=$ ______________________ Rational or irrational?

   (d) $\sqrt{11}\cdot\sqrt{11}=$ ______________________ Rational or irrational?

6. Based on #5, classify the following statement as true or false:

   Statement: The product of two irrational number is always irrational. True or False

7. Let's explore adding rational numbers. Using what you learned about in middle school, add each of the following pairs of rational numbers by first finding a **common denominator** then combine. Then, determine their repeating or terminating decimal.

   (a) $\frac{1}{2}+\frac{2}{3}=$ (b) $\frac{3}{4}+\frac{1}{2}=$ (c) $\frac{3}{8}+\frac{5}{12}=$

   (d) Classify the following statement as true or false:

   Statement: The sum of two rational numbers is always rational. True or False

8. Finally, what happens when we add a rational and an irrational number (we explored this in Exercises #4 through #6 in the lesson). Fill in the blank below from what you learned in class.

   The sum of a rational number with an irrational number will always give a(n) ________________ number.
   rational   irrational