Name: ____________________________________ Date: __________________

EXPONENTIAL MODELS BASED ON PERCENT GROWTH
COMMON CORE ALGEBRA I

There are many examples of growth in the real world that occur at a **constant percent rate**. These phenomena give rise to exponential functions. These functions will be easy to build and understand if you felt comfortable with the last lesson on percent growth and decay.

***Exercise* #1:** A population of fruit flies is growing at a constant rate of 6% per hour. The population starts, at $t=0$, with 28 flies.

(a) Using what we learned in the last lesson, determine the population after each of the following amounts of time. Show the calculation you use as repeated multiplication.

$t=1$ hr $\quad P=$

$t=2$ hr $\quad P=$

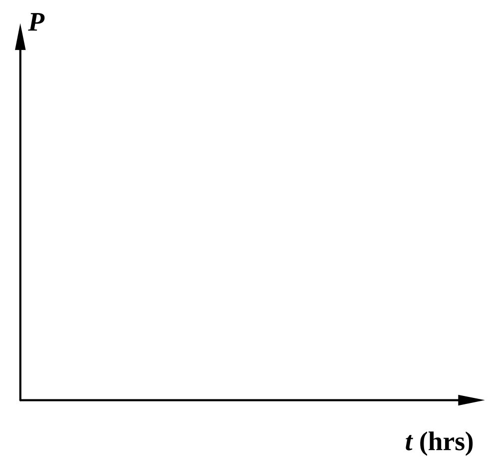
$t=3$ hr $\quad P=$

(b) Based on (a), find a formula that models the population, *P*, as a function of the time in hours, *t*.

(c) What is the value of $P(24)$?

(d) What does the calculation you made in (c) represent about the fly population? State the **range** of the population function over the **domain** interval $0 \le t \le 24$.

(e) Using your calculator, sketch a graph of this function over the interval $0 \le t \le 24$ and $0 \le P \le 120$. Mark the *y*-intercept.



Exponential growth is fairly easy to model and fairly easy to interpret in the model. Consider the following example.

***Exercise* #2:** If the savings in a bank account can be modeled by the function $S(t)=250(1.045)^t$. Which of the following is true?

(1) The initial amount deposited was \$250 and the interest earned is 45%.
(2) The initial amount deposited was \$2.50 and the interest rate is 4.5%.
(3) The initial amount deposited was \$250 and the interest rate is 4.5%.
(4) The initial amount deposited was \$2.50 and the interest rate is 45%.

We should also be able to model exponentially **decreasing** phenomena based on what we learned in the last lesson about percent decrease. Remember to always model based on **the percent that remains**.

***Exercise* #3:** As water drains out of a pool, the depth of the water decreases at a constant percent rate of 20% per hour. The depth of the water, when the draining begins, is 12 feet.

(a) As in #1, find the depth, D, of the water in the pool after each of the following times, t.

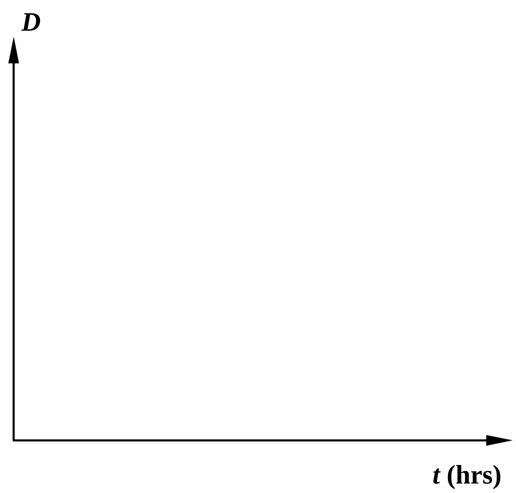
$t=1$ hr $D=$

$t=2$ hr $D=$

$t=3$ hr $D=$

(b) Based on (a), create an equation that gives the depth, D, of the water in the pool as a function of the time in hours it has been draining, t.

(c) Using your calculator, sketch a graph of your function over the interval $0\le t\le 20$ and $0\le D\le 15$. Mark the y-intercept with its value.



(d) It's safe to cover the pool after it reaches a depth of 1 foot or less. What is the minimum number of whole hours that we should wait to cover the pool? Explain how you found your answer.

***Exercise* #4:** Which of the following equations would model an exponential quantity that begins at a level of 16 and decreases at a constant rate of 8% per hour?

(1) $Q=16(0.92)^t$ (3) $Q=16(1.08)^t$

(2) $Q=16+0.92^t$ (4) $Q=16(-7)^t$

***Exercise* #5:** If \$350 is placed in a savings account that earns 3.5% interest applied once a year, then how much would the savings account be worth after 10 years?

(1) \$522.88 (3) \$472.50

(2) \$426.34 (4) \$493.71

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EXPONENTIAL MODELS BASED ON PERCENT GROWTH
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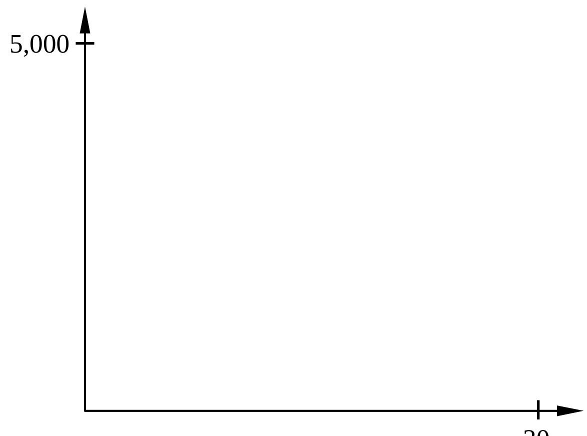
APPLICATIONS

1. An oil spill is spreading such that its area is given by the exponential function $A(t) = 250(1.15)^t$, where A is the area in square feet and t is the time that has elapsed in days.

(a) How large was the oil spill initially, i.e. at $t = 0$?

(b) By what percent is the oil spill increasing each hour?

(c) Sketch a graph of the area of the oil spill over the interval $0 \le t \le 30$ and $0 \le A \le 5000$ using your calculator. Label the *y*-intercept.



(d) After how many days will the oil spill reach a size of 3,000 square feet? Round to the nearest tenth. Solve graphically using the **INTERSECT COMMAND** on your calculator.

(e) What is the average rate of change of $A(t)$ over the interval $0 \le t \le 10$? Include proper units and don't round.

2. If a flock of ducks is growing by 6% per year and starts with a population of 68, how many ducks will be in the flock after 10 years?

(1) 109 (3) 122

(2) 198 (4) 408

3. A bank account earns interest at a rate of 3.5% per year (in other words it increases in value by that percent) and starts with a balance of $350. Which of the following equations would give the account's worth, *W*, as a function of the number of years, *y*, it has been gaining interest?

(1) $W = 350(1.035)^y$ (3) $W = 1.035y + 350$

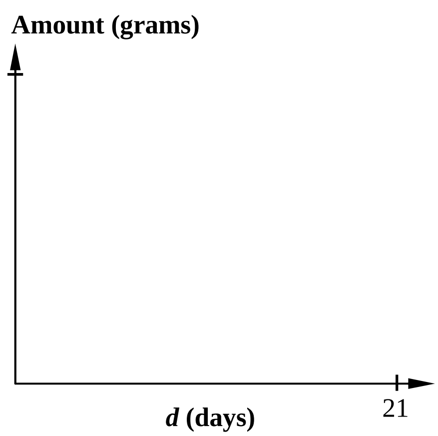
(2) $W = 350(0.35)^y$ (4) $W = 1.35y + 350$

4. The amount, A, in grams of a radioactive material that is decaying can be modeled by $A(d)=450(0.88)^d$, where d is the number of days since it started its decay.

(a) By what percent is the material decaying per day?

(b) Give an interpretation of the fact that $A(14)=75$.

(c) Use your calculator to sketch a graph of $A(d)$ over the interval $0 \le d \le 21$. You determine an appropriate y-window and label the y-intercept with its value.



(d) The material is safe to transport once it has less than 5 grams of radioactive mass left. Using tables on your calculator, determine the first day when it will be safe to transport this material. Show some entries from your table to support your answer.

x	Y_1

5. Newton's Law of Cooling can be used to predict the temperature of a cooling liquid in a room that is at a certain steady temperature. We are going to model the temperature of a cooling cup of coffee. The Fahrenheit temperature of a cup of coffee, T, in a room that is at a 72°F is given as a function of the number of minutes, m, it has been cooling by:

$$T(m)=114(0.86)^m+72$$

(a) Find $T(0)$ and using proper units, give a physical interpretation of your answer.

(b) What does the coefficient of 114 represent in terms of the situation being modeled?

(c) By what percent does the difference between the temperature of the coffee and the temperature of the room decrease each minute?

(d) I like my coffee when it is a nice temperature of around 100°F. How long should I wait?